2. Homework Assignments

Dynamical Systems II

Bernold Fiedler
http://dynamics.mi.fu-berlin.de/lectures/

due date: Thursday, October 30, 2014

Problem 1: Let $I \subset \mathbb{R}$ be an open interval and $A \in C^1(I, \mathbb{R}^{n\times n})$.
Prove: If A and $\dot{A}$ commute, i.e. if $A(t)\dot{A}(t) - \dot{A}(t)A(t) = 0$ for all $t \in I$, then

$$\frac{\mathrm{d}}{\mathrm{d}t} e^{A(t)} = \dot{A}(t)\, e^{A(t)} = e^{A(t)}\, \dot{A}(t).$$

Show, that if $A(t)$ and $\int_0^t A(\tau)d\tau$ commute for all $t \in \mathbb{R}$, the Wronskian $W(t,0)$ of the equation $\dot{x} = A(t)x$ has the form $W(t,0) = \exp\left(\int_0^t A(\tau)d\tau\right)$.

Problem 2: Consider the iteration of the real map $f(x) = ax(1-x^2)$ for positive parameter a and $x \in \mathbb{R}$.

(i) Determine the fixed points and their stability.

(ii) What happens at the parameters a=1, 3/2, 2, respectively?

(iii) Explore the behavior near these parameters via some numerical plots of the iteration.

Problem 3: Consider the partial differential equation $u_t = u_{xx} + f(u)$, where indices indicate partial derivatives of solutions $u = u(t,x)$. A travelling wave with wave speed c is a solution of the special form $u(t,x) = U(x - ct)$. Derive an ordinary differential equation for U.
Now consider cubic nonlinearities $f(u) := u(1-u)(u-a)$, for any fixed parameter $0 < a < 1$. Show that $U(\xi) := 1/(1 + \exp(-k\xi))$ is a traveling wave, for suitable k. Determine the wave speed c of the traveling wave U.

Problem 4: Consider the pendulum $\ddot{x} + f(x) = 0$ with odd nonlinearity $f \in C^1$ and $f' \geq 1$. Let $p(a) > 0$ denote the minimal period with amplitude $a > 0$.
Prove or disprove: The trivial Floquet multiplier 1 is:

(i) always algebraically double

(ii) geometrically simple for $p'(a) \neq 0$

Problem 5: **[Extra Exercise for non-mandatory extra points]** Prove or disprove the following claim. Consider the nonautonomous linear system $\dot{y}(t) = A(t)y(t)$ with the periodic real matrix A, $A(t + p) = A(t)$. Then there exists a decomposition of the Wronskian $W(t, 0) = Q(t)e^{Bt}$ where B is a real matrix and Q is a $2p$ - periodic function with $Q(0) = \text{id}$.